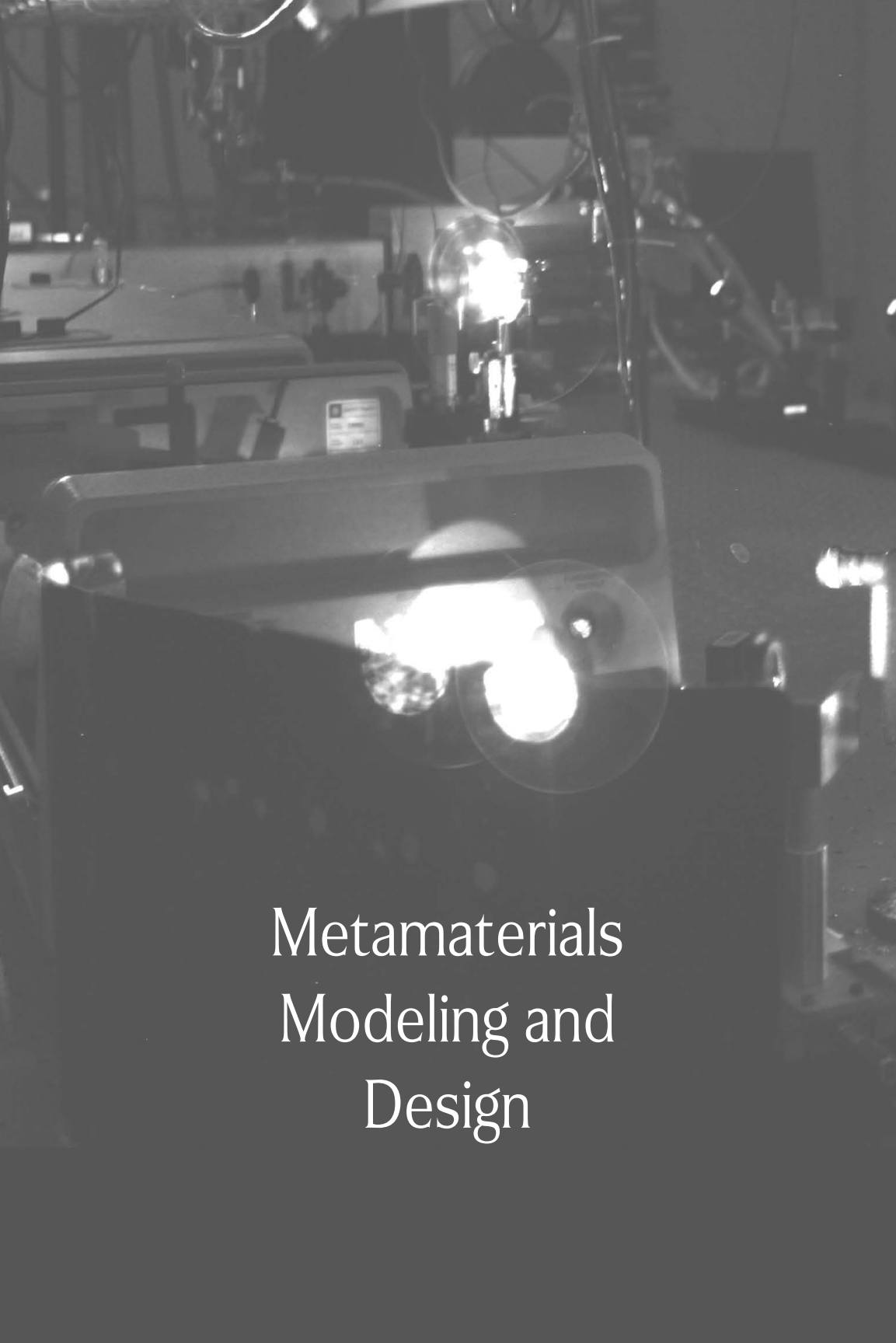


edited by

Didier Felbacq | Guy Bouchitté

Metamaterials Modeling and Design





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Preface

The domain of metamaterials now covers many areas of physics: electromagnetics, acoustics, mechanics, thermics, and even seismology. Huge literature is now available on the subject but the results are scattered. Although many ideas and possible applications have been proposed, which of these will emerge as a viable technology will only unfold with time. This book is concerned with electromagnetic waves only and deals essentially with the hard science, mathematical and numerical, behind the often spectacular, but somewhat oversold, possible applications of metamaterials. In a rapidly evolving field, with lots of would-be revolutions, spending too much pages on the zoology of metamaterials would certainly condemn this book to a rapid obsolescence. By contrast, the theoretical and numerical methods presented here are the basis upon which future trends will be built.

The first chapter is a survey of Maxwell's equations and their main properties. After a short historical introduction, potentials and conservation laws are addressed. Then comes a brief presentation of the formulation of Maxwell equations using differential forms. Finally, causality and its consequences are addressed.

Chapter 2 provides the elements of the physics of materials required to bridge semiconductor and metal sciences with electromagnetism, and Chapter 3 is a general reflection upon the notion of averaging and the definition of effective properties.

Chapter 4 is a crash course on basic principles of transformation optics. Simple examples in cylindrical geometry are given using radial transformations that show the unifying power of the concept: mapping an open domain on a bounded domain, perfectly matched layers, invisibility cloaks, and superlenses. Some numerical

simulations are presented as an illustration including cloaks of arbitrary shapes and mimetism.

Chapter 5 is concerned with wave propagation in periodic media. The theory of Bloch waves is described in detail. The situation where the medium does not cover the entire space is addressed, because in that situation the boundary of the periodic medium is decorated with evanescent modes. Evanescent waves are then investigated. They are shown to be a complexification (in the mathematical meaning) of the Bloch spectrum.

Chapter 6 tackles the problem of diffraction of electromagnetic waves by a bidimensional grating. A new formulation based on finite element method is proposed. A lot of academic cases and more challenging cases are given for highlighting both the versatility and the powerfulness of the method described in this chapter. The second part of the chapter is devoted to the method of multiple scattering, which is presented for a collection of parallel cylinders.

Chapter 7 is the first chapter devoted to effective properties of metamaterials. Periodic structures are considered and the period of the materials are small with regard to the wavelength of the incoming wave. Besides, the materials are supposed to be of low contrast: this is the framework of soft problems. Closed formulae are given in some academic cases such as small spherical and small circular cylindrical inclusions. A special attention is drawn on spherical inclusions and the mixing formulae (Rayleigh, Maxwell Garnett, Bruggeman) are compared to the two-scale theory.

Chapter 8 addresses the homogenization of highly contrasted objects. The first situation investigated is that of a periodic collection of thin metallic wires. It is shown that the effective medium obtained is dispersive and has a plasmonic resonance. In the second part, the theory is extended to deal with finite-length rods. It is proven that the effective medium becomes spatially dispersive. The chapter closes with numerical investigations of the properties of the effective medium.

The final chapter is also devoted to homogenization theory. It deals with the possibility of homogenizing metamaterials for

frequency above the first band and taking into account the Mie resonances. Bidimensional resonant dielectric metamaterials are addressed and the onset of an effective magnetic activity is proven.

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