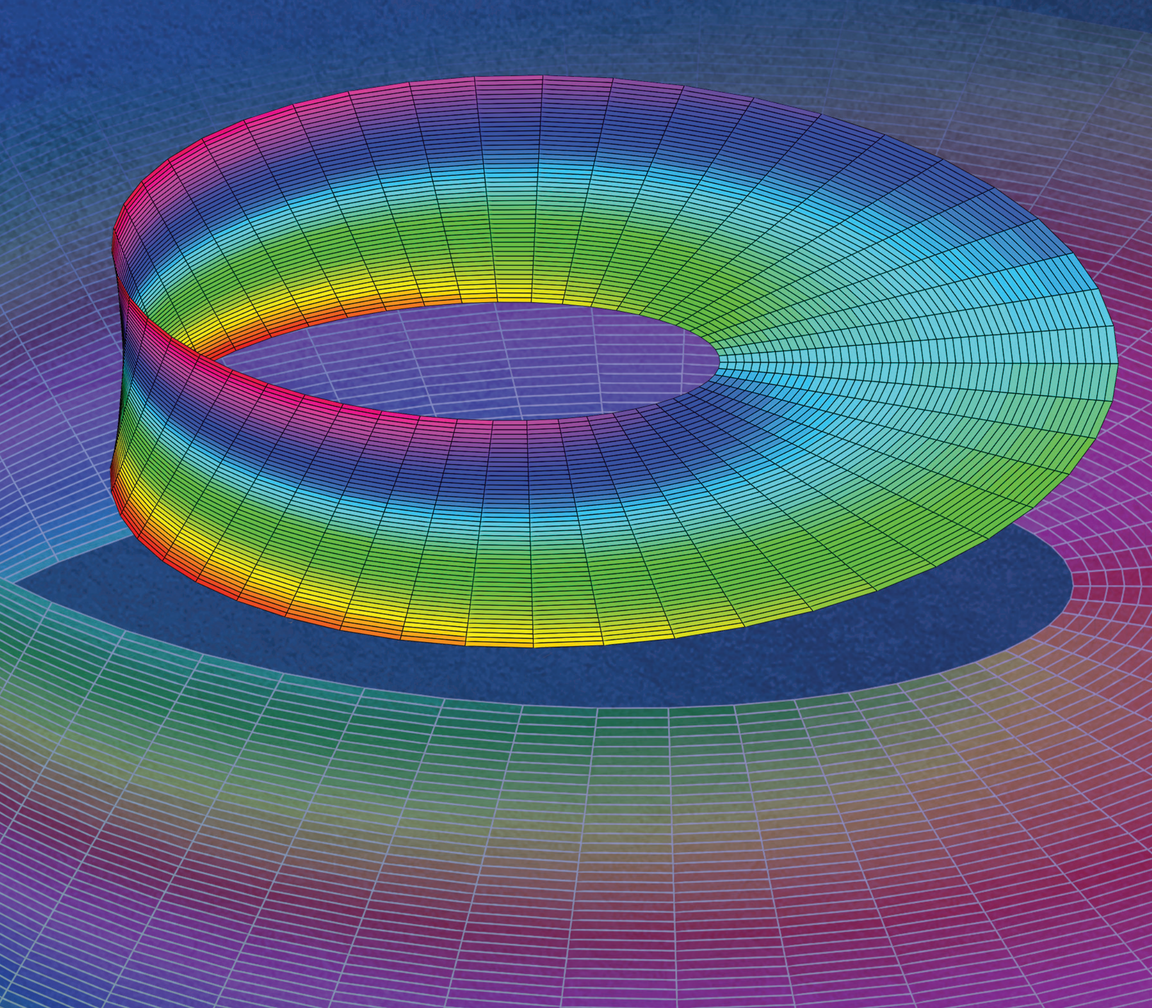
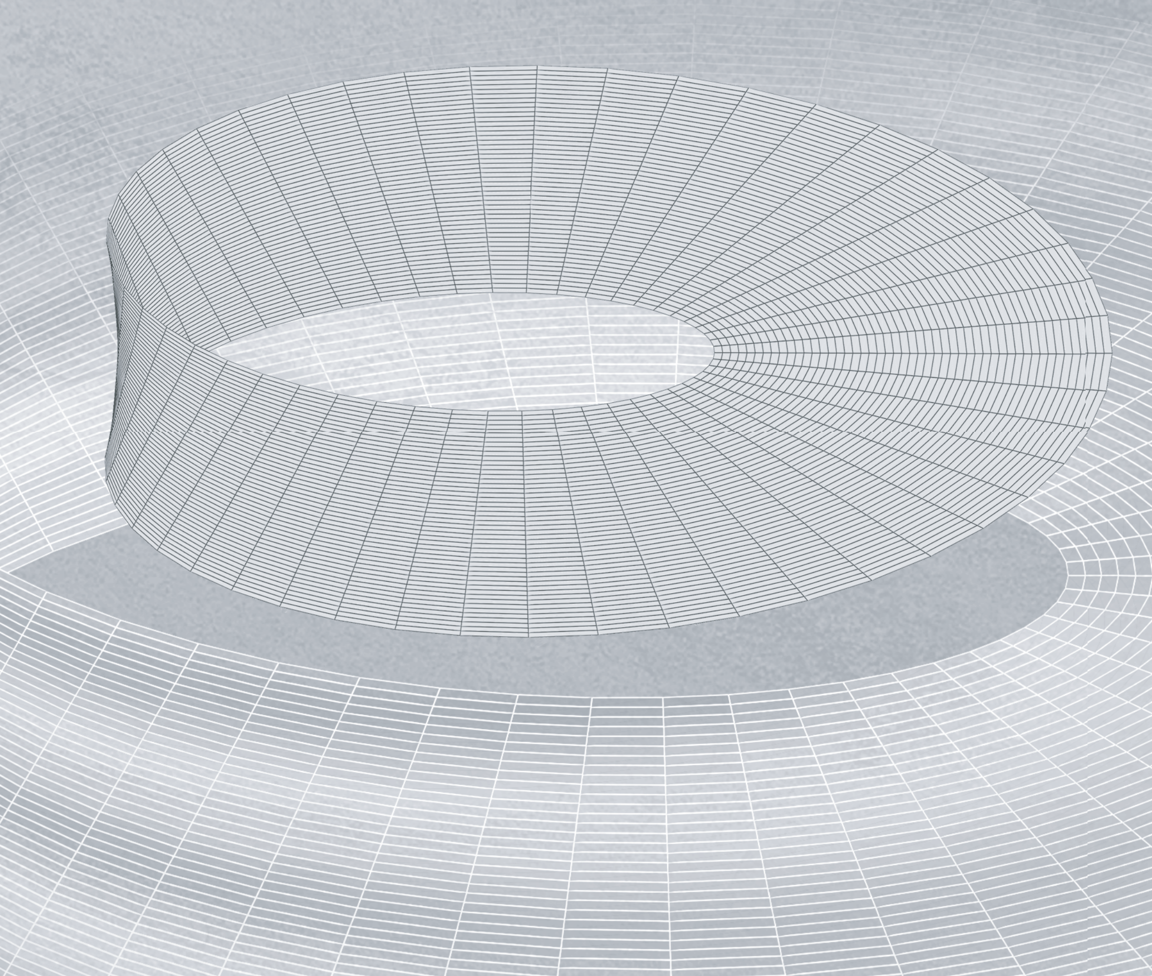


The Möbius Strip Topology

History, Science, and Applications in Nanotechnology, Materials, and the Arts

Klaus Möbius | Martin Plato | Anton Savitsky



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Contents

Acknowledgments

Since June 2015, when our correspondence with Jenny Stanford Publishing (formerly Pan Stanford Publishing) started with the invitation to write a book on Möbius Strip Topology based on our magnetic-resonance research on Hückel–Möbius aromaticity switching in chemistry, the content of the book has substantially extended: It should now include Möbius topology applications also in other fields of science, in the arts and architecture as well as give historical perspectives. We are grateful to Ms. Jenny Rompas and Mr. Stanford Chong from Jenny Stanford Publishing for having encouraged us for this extension and their continuing support for more than 5 years, which we finally needed to realize this book project.

The historical parts of the book naturally include biographical observations about our protagonist, August Ferdinand Möbius (1790–1868), a distinguished mathematician and astronomer in Leipzig. We also describe the time in which he lived in. We are particularly thankful to André Loh-Kliesch, Leipzig, for having sent us his PhD dissertation (*August Ferdinand Möbius, Leben und Werk*, Universität Leipzig, 1994) as an authentic and often unique historic source for Möbius's biography. Decisive periods of his life were marked by the Napoleonic Wars in Central Europe with subsequent wars of liberation and social revolutions. After initial successes, the democratic uprisings were put down with military force. What followed was the restauration period of the absolutist monarchies with their wide-ranging censorship and suppression of the request for democratic freedoms. All this had serious consequences also for the development of sciences and societies in Central Europe. For the assessment of these historical consequences in mathematics, André Loh-Kliesch's dissertation was very helpful. We acknowledge with gratitude the scientific cooperation with Lechosław Latos-Grażyński and his coworkers (University of Wroclaw) who were the first to synthesize expanded porphyrin molecules with Möbius–Hückel topology switching capabilities—as we describe in one of the Case studies of the book. We thank Takeji Takui (Osaka City University) for his help in the evaluation of a popular Japanese novel whose plot follows the structure of a Möbius strip with a twist of the characters as they move through the loop of the story. We are indebted to Lorenz Haarmann (Berlin) for gifting us his magnificent photos of the Phoenix Media Center in Beijing, which was recently built in the form of a giant Möbius–Strip torus.

We are grateful to Wolfgang Lubitz (Max Planck Institute for Chemical Energy Conversion, Mülheim/Ruhr) and Alexey Semenov (Lomonosov State University Moscow), Giovanni Venturoli and Paola Turina (University of Bologna), Kev Salikhov (Zavoisky Physical-Technical Institute, Kazan) as well as Joachim Marcus-Steiff (Paris) and Fritz Franzmeyer (Berlin) for their critical and constructive comments on several parts of the manuscript. Their encouragement over the years was indispensable for us to continue. We thank Georg Möbius (Neuss) for his thorough genealogical research on Möbius families, which has not discouraged us. We also thank Uta Möbius (Berlin), Katharina Möbius (Winnemark), and Janina Möbius (Berlin) for their insisting, but always well-meaning questions about the connection of the presented book sections with the general topic of the book. In doing so, they repeatedly forced us to give a convincing rationale for our choices. Thereby, they have protected us from being carried away by our own enthusiasm for what we had written. This has certainly been to the benefit of coherence of the book.

Preface

The intention of this *Preface* is to give some latest news headlines from 2021, a *postscriptum* of our book in a way, which are related to August Ferdinand Möbius (1790–1868) and his heritage as mathematician and astronomer.

For more than one and a half thousand years, long before the discovery of the "Möbius strip" as a topological figure with a one-sided surface in the 19th-century mathematics, the mysterious symmetry behavior of half-twisted ribbons has played a role in Western and Eastern cultures as a symbol of unity despite diversity. Recently, the Möbius strip has received an unexpected actualization and visibility as the logo of the European Union (EU) Council Presidency by Germany on July 1, 2020, for a period of 6 months (see Fig. 1). Germany took over the Presidency in the midst of the first global wave of the Corona Covid-19 pandemic. The EU logo designed for this period, a Möbius strip representation, symbolizes an "inclusive and innovative Europe in which the most diverse people and interests come together to form a common entity," according to the official statement by the German government.

August Ferdinand Möbius was not only an outstanding mathematician but also a renowned astronomer. For example, in February 1822, Möbius observed for a month the opposition of Mars and Earth. This was a real astronomical spectacle: During the orbits of planets Earth and Mars around the Sun, their different orbiting velocity leads to the "opposition" situation in which the two planets are on opposite sides of the Sun. During opposition, Mars and Sun are on directly opposite sides of Earth. From our perspective, Mars rises in the East just as the Sun sets in the West. Then, after staying up in the sky the entire night, Mars sets in the West just as the Sun rises in the East.

August Ferdinand Möbius would certainly have liked to receive the following spectacular news of planet Mars from the future, 199 years ahead:

Figure 1 *Left*: Möbius-strip logo of the European Union Council Presidency by Germany from July 1 to December 31, 2020. It is a symbol of unity and connectedness. Because even if you start on the Möbius strip from different sides and in different directions, you will still meet on the same side: there exits only one side. The EU member states have voluntarily associated themselves across all differences, national interests, and disagreements to ultimately move forward together on the democratic European course. In the Presidency Program "*Together for Europe's recovery*", the focus is put on overcoming the Covid-19 pandemic and seeking answers to the challenges of the climate change mitigation. *Right*: Angela Merkel, from November 22, 2005, to December 8, 2021, Chancellor of the Federal Republic of Germany, in times of Corona pandemic.

After a 480-million-kilometer flight to Mars, the lander vehicle carrying the *Perseverance* rover unit together with its small *Ingenuity* helicopter was announced to have successfully landed in *Jezero Crater* of Mars at 20:55 UTC on February 18, 2021; the landing signal from Mars took 11 min to arrive at Earth. The landing of the *Perseverance* rover on Mars is part of the "Mars 2020 Mission" by NASA; it is NASA's fifth Mars rover mission in the frame of the *Mars Exploration Program*. Using an Atlas V launch vehicle, the Mars lander unit was launched from Cape Canaveral on July 30, 2020.

Perseverance was designed to study Mars rocks in detail with respect to bio-signatures, geological processes, and the planet's geological history in order to gain, among other things, insights into possible earlier life on Mars (see Fig. 2).

August Ferdinand Möbius's legacy in mathematics and astronomy has turned into our daily experience of science and technology. Therefore, we focus the present book on typical examples of current activities in science, art, and architecture; they are examined against the background of political events and social and cultural tendencies.

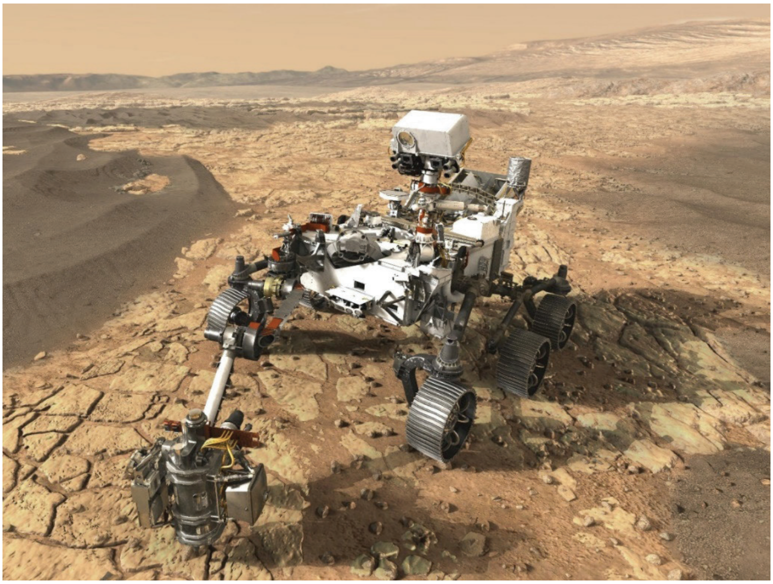
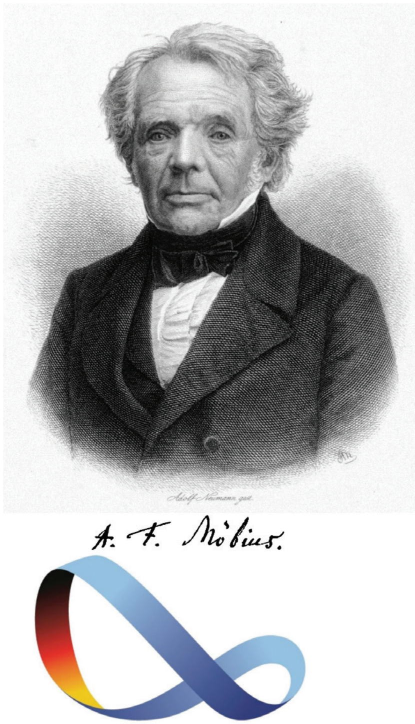


Figure 2 NASA's newest robotic explorer *Perseverance* landed safely on Mars in the *Jezero Crater* on February 18, 2021, ready to begin seeking for signs of ancient life. The approach and landing maneuvers of the *Perseverance* on Mars had to be fully automated in advance because of the long signal propagation time of about 11 min between Earth and Mars. The landing stage was able to detect any unforeseen obstacles that might appear on the surface of Mars and, if necessary, to change the landing site with sideways movements of up to 300 m. *Perseverance* runs on nuclear power batteries because on Mars, global dust storms render solar panels useless. (Image Credit: NASA/JPL-Caltech).

Klaus Möbius
Martin Plato
Anton Savitsky
Summer 2022

Introduction



August Ferdinand Möbius (1790–1868) and the half-twisted Möbius loop: a topological paradigm of one-sided surfaces

> “*What makes a great mathematician? A feel for form, a strong sense of what is important. August Ferdinand Möbius had both in abundance. He knew that topology was important. He knew that symmetry is a fundamental and powerful mathematical principle.… His mathematical taste was imaginative and impeccable. And, while he may have lacked the inspiration of genius, whatever he did he did well, and he seldom entered a field without leaving his mark.… No body of deep theorems, but a style of thinking, a working philosophy for doing mathematics effectively and concentrating on what’s important. That is Möbius’s modern legacy. We couldn’t ask for more.*”

> (Ian Stewart in John Fauvel et al., *Möbius and His Band*, Oxford University Press, 1993). Ian Stewart (born 1945) is a British mathematician at the University of Warwick, UK. He received the prestigious Michael Faraday Prize of the Royal Society in 1995 for his exceptional contributions to the popularization of mathematical sciences).

It is *Möbius's modern legacy* that has fascinated us during the years of doing research for our book project "*The Möbius Strip Topology: History, Science, and Applications in Nanotechnology, Materials, and the Arts*." While doing this work, we concluded that we should not try to write a biography about the mathematician August Ferdinand Möbius, a former pupil of Gauss, in the traditional manner, in which a historic account of the protagonist's life from birth to death is presented chronologically, with a grouping of noteworthy events around the main character. Rather, we should report on August Ferdinand Möbius in the mirror of his time with its political and social peculiarities—and in the context of his contemporaries, especially Carl Friedrich Gauss and Alexander von Humboldt, whom he admired and who strongly influenced him. Who played a major role in creating an intellectual environment for the abstract sciences such as mathematics and astronomy in Germany and beyond, in which intellectual achievements like topology were produced. In fact, Möbius's legacy reaches into today's modern sciences and arts and is being rediscovered time and again. Thus, we consider August Ferdinand Möbius not so much as the sole protagonist of our book, but rather as a representative of civil society in the period between the 18th and 19th centuries when a fundamental change took place in the political, scientific, and cultural world of Central Europe.

The one-sided Möbius strip (or band) is a paradigmatic example of the fascination and significance of mathematical topology for understanding unexpected consequences of complex object formation. In 2021, the Möbius strip has its 160th birthday, i.e., it is not really a young object, but still going strong in its vibrancy. The birth of the Möbius strip was accompanied by the birth of topology (from the Greek τόπος, place, and λόγος, study); this branch of mathematics is concerned with the invariants of objects that are preserved under their continuous deformations such as stretching, crumpling, and bending.

Over the years, topology grew and grew and became an important branch of abstract mathematics, with significant impact in almost all areas of the sciences and humanities. The study of topology does not only introduce new concepts and theorems in abstract mathematics but also creates fascinating practical applications of strange symmetry aspects for weird geometrical objects and underlying principles, for instance in science, art, and architecture.

For the preservation of August Ferdinand Möbius's scientific legacy, the Saxon Society of Sciences had established a Möbius archive in Leipzig, which contained Möbius's rich correspondence with other mathematicians and astronomers. Sadly, this archive was burned in 1943 due to the effects of World War II, actually, by allied area bombing. A detailed appreciation of the scientific work of A. F. Möbius is given (in German) in the doctoral dissertation of André Loh at the University of Leipzig from the year 1994, entitled "*August Ferdinand Moebius (1790–1868) – His life and work*" [1]. In this excellently researched biography of Möbius, the author discusses in detail his major works, books, and essays, as well as Möbius's influence on the development of astronomy and mathematics. Many testimonies on polyhedra, the fundamental geometric object for the development of topology, were first compiled by Johann Benedict Listing, another pupil of Gauss, in his "*Der Census räumlicher Complexe oder Verallgemeinerung des Euler'schen Satzes von den Polyedern*" (A census of spatial complexes, or a generalization of Euler's theorem on polyhedra) (1861), as well as somewhat later but independently, by August Ferdinand Möbius in his "*Über die Bestimmung des Inhalts eines Polyeders*" (On the determination of the volume of a polyhedron) (1861, 1865).

The dissertation of André Loh is the first comprehensive Möbius biography describing the life as well as the scientific oeuvre of the eminent Leipzig astronomer and mathematician; hereby it combines socio-historical and ideas-historical approaches to the subject. In doing so, the strong influence of adverse external life circumstances caused by the Napoleonic Wars and the subsequent political reorganization of Central Europe on the scientific work of a scholar in the first half of the 19th century is presented in a comprehensive and convincing manner. Loh's evaluation of archival documents and publications that have not been addressed so far provides a number of revealing details, which are highly significant for the historical positioning of August Ferdinand Möbius in his time.

The description of Möbius's life and work by André Loh follows his academic *Curriculum Vitae* from a student of a Prince College in Schulpforta (near Naumburg at the river Saale), to a full professor at the University of Leipzig. It also covers his more than 50 years of activity as a researcher and teacher in his various periods of creativity. There is considerable coverage of Möbius's family and collegial environment, which was formative for him over the years in crucial stages of life.

As you would expect, the book we are presenting is not the first book *in English* on the history of August Ferdinand Möbius and his topological masterstroke, what is now called the Möbius strip (or band). It is, in fact, the third book in English to sketch his contributions to mathematics and astronomy, all embedded in 19th-century Central Europe's history, society, and science. Specifically, we refer here to the two earlier books [2, 3].

Actually, there is still another earlier book (2013) on the history of August Ferdinand Möbius; it was written *in French* by the French historian of geometry Dominique Flament, and the title is "*August Ferdinand Möbius: Entre polyèdres et corrélation élémentaire*" (August Ferdinand Möbius: Between polyhedra and elementary correlation) [4]. In his elaborate study, Flament examines the evolution of the idea of the polyhedra from the Platonic solids to the Möbius band. He summarizes that when looking at the transformations the polyhedron has undergone since its first state as a "Platonic solid," it can be seen that the Möbius strip actually belongs to the great conceptions of the theory of polyhedral and leads to the "topological toy" that still fascinates us today.

The previous books by Fauvel et al. and Pickover are still great reading experiences, which have lost nothing of their charm and topicality since the years of their publication (1993 and 2006, respectively). On the other hand, in the meantime, a large number of modern research results and breathtaking applications of Möbius strip topology in chemistry, physics, and even in art and architecture have appeared, either in specialized journals or in public media, and we have tried to include representative examples of them to a certain detail in our book.

Furthermore, we studied in a comprehensive framework of historical facts the political conditions in Central Europe of the 19th century, where and when our protagonists lived. Over the last 5 years, we have carried out intensive Internet research in international historical encyclopedias, libraries, and archives, made virtual and real local museum visits, where traces and testimonies of August Ferdinand Möbius and his contemporaries in science are still preserved and accessible, to elucidate the details of the historical processes and try to understand the actions (or nonactions) of our protagonists. Unfortunately, since 2020, this historical source search has become much more difficult by the global Corona pandemic with the subsequent lockdowns of public institutions. Generally, their websites in the Internet remained accessible; they often reveal, however, only the standard, mostly rather patriotic, views of the 18th- and 19th-century kings and emperors, army generals, and field marshals. In rare cases, they also contain descriptions of the suffering of the local people under the Napoleonic Wars: With the unimaginable cruelties of the soldiers of the mercenary armies, gigantic death tolls and destructions, particularly in the central German states Saxony and Prussia, where many characters of our book lived.

In view of the impression we got from our historical research, we were wondering why this specific 19th-century period of Napoleonic Wars and forced invasion—and the concomitant renunciation of all humanistic values—was so little reflected in the self-testimonies of the mathematical elites in Germany of that time, first and foremost by Carl Friedrich Gauss and his student August Ferdinand Möbius. Möbius's academic education and subsequent academic career took place in Leipzig, in the immediate vicinity of the battlefields of the Napoleonic Wars, like those of Jena-Auerstedt (1806) and of the Battle of Nations (1813) near Leipzig.

These questions and many facets of the historical events in the period from 1790 to 1868, in which August Ferdinand Möbius's academic education and his academic career took place, are explored in Chapters 2, 3, and 4.

The historical sources are full of appreciations of the resurrection of the German universities and pure sciences, in particular abstract mathematics, after the Napoleonic regime of war and subjugation. The rise largely occurred as a result of the acclaimed Stein–Hardenberg reforms of the civil societies in Prussia and other German states. On the cultural level, they were initiated and assisted by then very popular Wilhelm von Humboldt and his younger brother Alexander von Humboldt. The Stein–Hardenberg reforms, introduced in the years 1807–15, created the basis for Prussia's transformation from an absolutist and agrarian state to an enlightened national and industrial state.

The 18th and 19th centuries are often viewed as the centuries of revolutions. In America, 1776 marks the end of the American revolution. In France, 1789 marks the end of the French Revolution, and soon after, the period of the Napoleonic regime is heralded. Napoleon Bonaparte (1769–1821) temporarily gained dominion over large parts of continental Europe, enforced by his mercenary *Grande Armée*.

In the German states and Austrian Empire, 1848 marks the revolutionary events of the liberal, bourgeois-democratic and national rebellions for national unity and independence and against the restructuring efforts of the Holy Alliance, to which the ruling houses in large parts of Central Europe had allied themselves from 1815 onward (Congress of Vienna). From Berlin to Vienna, the 1848 Revolution forced the appointment of liberal governments in the individual German states and the conduct of elections to a constituent National Assembly. It convened on May 18, 1848, in the *Paulskirche* in the then Free City of Frankfurt (Main).

By July 1849, the first attempt to create a democratically constituted, unified German nation-state was put down by the Holy Alliance with military force, predominantly by Prussian and Austrian troops. The subsequent persecution of supporters of a liberal, republican-democratic or socialist ideology caused tens of thousands to flee the German states in the years after 1848–49. They initially found asylum mainly in France, England, or Switzerland.

These political and social revolutions in Central Europe are to be seen in the context of the global Industrial Revolution where scientific methods were being applied to increase productivity and life standard. Both France and Germany were caught up in the age of revolution, but the two countries reacted quite differently. The 18–19th century period saw an impressive rise in the sciences and technologies. For example, an increase in breadth and complexity of mathematical concepts unprecedented in the history of mathematics.

After the French Revolution, Napoleon emphasized the practical usefulness of mathematics in times of war and preparations of new ones. And his military ambitions gave French mathematics a big boost in Applied Mathematics, as exemplified by the giants Lagrange, Laplace, Legendre, and Fourier.

Germany, after the crushing defeat of Prussia and its allied German constituent states by Napoleon's *Grande Armée* at the double battle of Jena and Auerstedt in 1806, took a different approach to implementing pressing scientific and cultural reforms. Under the influence of the great humanist and cosmopolitan scholars Wilhelm and Alexander von Humboldt, Germany was supporting, for example, pure learning and research in pure mathematics for its own sake, basically detached from the demands of the military leaders for extended applications of mathematics in modern armament development. Through Humboldt's initiatives, the Berlin University was founded in 1810. Even today, this university is considered the "mother of all modern universities." Wilhelm von Humboldt advocated a "*Universitas litterarum*" in which the unity of teaching and research would be realized and an in-depth humanistic education for students would be made possible. This idea proved successful, spread worldwide, and gave birth to many universities of the same type in the following century and a half.

Separated from the *Universitas litterarum*, in the States of the German-speaking Europe, technical and military education and research continued to take place in the first decades of the 19th century in a large number of trade schools and polytechnic colleges, some of which are still important and significant today (including Berlin in 1821, Karlsruhe in 1825, Munich in 1827, Stuttgart in 1829, Hanover in 1831, Darmstadt in 1837). The underlying reasons for these foundations were, on the one hand, the requirements of the civil service and, on the other hand, the requirements of the developing industrial society. The tasks of the technical civil service had grown considerably; they were mainly in state mining, the military sector, building approvals, and from 1840 also railroad construction, which was nationalized in the course of the 19th century.

It was in the environment of the *Universitas litterarum* that the young Carl Friedrich Gauss, sometimes called the "Prince of Mathematics," worked at the prestigious University of Göttingen. Some of Gauss's ideas were a hundred years ahead of their time and touched on many different parts of the mathematical world, including geometry, number theory, calculus, algebra, and probability. He is widely regarded as one of the three greatest mathematicians of all times, along with Archimedes and Newton. Among his students was August Ferdinand Möbius.

August Ferdinand Möbius is best known for his 1858 discovery of the "Möbius strip," which became a paradigmatic subject of the then new mathematical discipline topology. As a side note on modernity, the importance of topology in today's science is still persisting, as was demonstrated, for example, when the Nobel Prize in Physics 2016 was awarded to David Thouless, Duncan Haldane, and Michael Kosterlitz "*for their theoretical discoveries of topological phase transitions and topological phases of matter*."

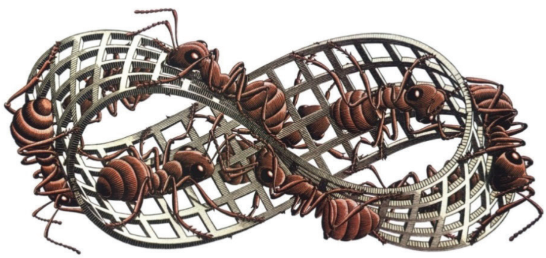


Figure 1 M. C. Escher, *Möbius Strip II (Red Ants)*, woodcut 1963. It shows ants crawling on the Möbius strip with the intention to traverse all parts of its (sweet) surface area. To their surprise, they are enjoying to crawl on all parts of the surface without ever having to cross an edge to the other side of the strip. The essence of one sidedness of the Möbius strip is that there is no "other side"; the irritated ants simply have to crawl far enough to go around twice before coming back to where they started. Mathematically translated, this means that on a Möbius strip a vector normal can be shifted to any point of the surface without ever crossing an edge.

Figure 2 Logos inspired by the Möbius strip with twisted loops (from *left* to *right*): Universal Recycling Symbol, Commerzbank, German Mathematicians Association.

It is fair to state that the concept of Möbius topology is fundamental in almost every branch of science, arts, and commerce. For instance, the topology of the Möbius strip with its one-sided surface (that requires twice as much paint to color as one might have thought) has inspired Maurits Cornelis (M. C.) Escher (1898–1972) to create his famed woodcuts and lithographs, among them is the 1963 woodcut *Möbius Strip II (Red Ants)* (see Fig. 1).

The public's appreciation of a personality of science or a scientific invention can sometimes be inferred from banknotes or stamps issued by the respective national bank or postal services. Figure 3 shows banknotes with the portraits of Leonhard Euler and Carl Friedrich Gauss, the two towering mathematical geniuses of the 18–19th century.

Figure 3 Banknotes (front). *Left*: Leonhard Euler, 10 Swiss Franc; *Right*: Carl Friedrich Gauss, 10 Deutsche Mark.

Figure 4 shows a selection of historical stamps from around the world that have a reference to mathematics and topology and their applications, as well as the scholars who left their deep marks in these fields.

Apparently, the Möbius strip is one of the few icons of mathematics that is been recognized by people from a broad variety of cultures. Its topological beauty is sensed even by laymen and inspired them to startling interpretations—also when designing, for example, furniture, fashion, and jewelry. Beyond such ephemeral creations, they are found in the enduring creative domains of music, literature, painting and sculpture, architecture, engineering, and science, all the way to dining culture.

Figure 4 International postal stamps with mathematicians involved in the early development of topology. Pierre-Simon Laplace (1749–1827): "*Read Euler, read Euler, he is the master of us all.*"

The nine chapters in this book cover a considerable range of subjects and characters. After the initial chapter, in which we give the timeline and historical background of science and cultural breakthroughs in Europe, we describe in Chapter 2 and Chapter 3 the time of August Ferdinand Möbius's early life and academic education (1790–1815) and his academic career (1815–68), respectively. Chapter 4 analyzes the reality of the Napoleonic Wars in Europe, the reality as it presented itself to the victims and eyewitnesses in the battle areas, for example in Saxony, where August Ferdinand Möbius lived. Chapters 5 to 8 are devoted to applications of Möbius strip topology in mathematics, astronomy, chemistry, as well as physics and nanomaterials. Finally, Chapter 9 highlights quite a few fascinating examples of Möbius strip topology from the arts and architecture.

Figure 5 *I'll never be able to finish this Möbius strip steak.* (Adapted from a cartoon by Dan Piraro, June 23, 2011. Source: Bizarro Comics. com).

If we take astronomy (Chapter 6) as an example, there was a revolution in astronomical instrument development, specifically in large high-resolution telescopes, which took place over the period of Möbius's lifetime and opened new horizons in quantifying stellar observations of the Milky Way galaxy. August Ferdinand Möbius at the Leipzig Observatory, for example, and his contemporary Friedrich Wilhelm Bessel at the Königsberg Observatory, depended on a small number of experienced physicists, optical experts, and precision mechanics, like Joseph von Fraunhofer (1787–1826) and Johann Georg Repsold (1771–1830), who built telescopes for them, which enabled observations of admirable numerical precision. In other words, the manufacturing technology of the instruments redefined our view of the universe. This point will be discussed in detail in Chapter 6.

Figure 6 (*Left*): Möbius strip chair (Takeshi Miyakawa, 2014. Born and educated in Tokyo, Takeshi Miyakawa is an artist based in Brooklyn, New York.) (*Middle*): Hand-knitted Möbius strip roundabout scarf that features a half-twist. (Möbius scarf knitting instructions are available on the Internet). (*Right*): Möbius strip necklace jewelry carved from Jade gemstone (designed and manufactured in China).

In the case of modern astronomy, the evolution of instruments to progressively larger telescopes became a major scientific and engineering project, which, due to the enormous costs involved, can only be coped with through international cooperation. This is especially true for space-based telescopes, for example the Optical and UV Hubble Space Telescope, which NASA and the ESA have been operating since 1990. It is named after the American astronomer Edwin P. Hubble (1889–1953), who confirmed that the universe is expanding, thereby providing the foundation for the Big Bang cosmological model of the origin of the universe. Another example is the Infrared Herschel Space Telescope, named after the German–British astronomer Wilhelm Herschel (1738–1822); it was operated from 2009 until 2013 by the ESA.

The Herschel Space Telescope sees infrared and sub-millimeter light, which can readily penetrate through the dust hovering between the bustling center of our galaxy and us. Herschel's detectors are also suited to see the coldest material in our galaxy. During and after operation, Herschel left a legacy of treasured data that are still being analyzed to produce a multitude of amazing scientific results every year.

For instance in September 2018, the ESA announced that a bizarre, twisted ring of dense gas and dust at the center of our Milky Way galaxy became visible from the analysis of the astronomical data that had been captured by the far-infrared cameras on board the Herschel Space Observatory (Fig. 7). Herschel's view reveals the entire ring for the first time. The nearly continuous strip of dense and cold clumps of material forms a loop figure that resembles either an infinity symbol (?), or a lateral number 8 (?), or a Möbius strip (?) that stretches across more than 600 light years of space. In this image, the strip twists around an invisible axis running roughly from the top left to the bottom right.

Figure 7 Astronomers of the "Herschel infrared Galactic Plane Survey" project using the Herschel Space Observatory have discovered that a suspected ring at the center of our galaxy is warped into an infinity-shaped loop. The image combines observations at three different wavelengths: 70 μm (blue), 160 μm (green), and 250 μm (red). The loop is estimated to have 30 million solar masses, and it is made up of dense gas and dust at a temperature of as low as 15° above absolute zero (such cold regions are coded in blue color). The loop and its surroundings harbor a number of star-forming regions and young stars that stand out in bright blue. This area is part of the Central Molecular Zone, a region at the center of the Milky Way permeated with molecular clouds, which are ideal sites for star formation. (Image credit: ESA/NASA/JPL-Caltech).

Chapter 7 is devoted to topology applications from chemistry. We analyze, for example, the concept of chemical stability versus instability of organic molecules on the basis of "Hückel aromaticity" (introduced in 1931 by Erich Hückel, 1896–1980) and "Möbius aromaticity" (introduced in 1964 by Edgar Heilbronner, 1921–2006). Both approaches are based on theoretical considerations of symmetry and topology of the electronic and geometrical structures of a molecular system with either planar or 180°-twisted skeletons (see Fig. 8).

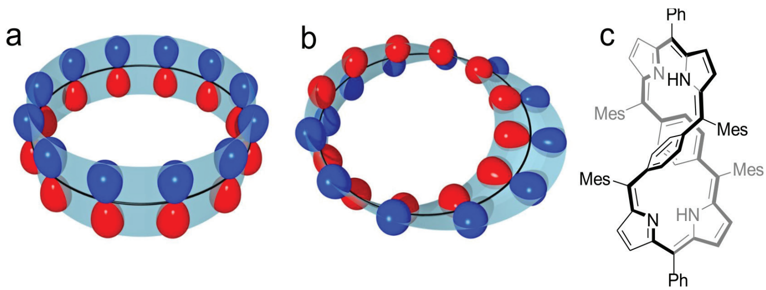


Figure 8 2p orbitals in aromatic molecules. Conventional planar (Hückel) 12-annulene, stable (**a**) and hypothetical 180° twisted (Möbius) 12-annulene, instable (**b**). Note the phase change by 180° between the two leftmost 2p orbitals in Möbius annulene (**b**). (Sources: P. B. Karadakov, Department of Chemistry, University of York, UK; D. L. Cooper, Department of Chemistry, University of Liverpool, UK; (**c**): Expanded porphyrin di-p-benzi[28]hexaphyrin(1.1.1.1.1.1), compound **1**, with Möbius topology (Mes = mesityl, Ph = phenyl). The 28-π-electron conjugation pathway is shown in bold. The expanded porphyrin was synthesized by L. Latos-Grażyński and coworkers at the University of Wroclaw, Poland [5].

In 2007, the group of Lechosław Latos-Grażyński at the University of Wroclaw, Poland, succeeded in synthesizing the compound di-p-benzi[28]hexaphyrin(1.1.1.1.1.1), which can dynamically switch between Hückel and Möbius conjugation depending, in a complex manner, on the polarity and temperature of the surrounding solvent. This discovery of "topology switching" between the two-sided (Hückel) and one-sided (Möbius) molecular state with closed-shell electronic configuration was based primarily on the results of NMR spectroscopy and DFT calculations. The present EPR and ENDOR work on the radical cation state of compound **1** is the first study of a ground-state open-shell system, which exhibits a Hückel–Möbius topology switch that is controlled by temperature, like in the case of the closed-shell precursor. The unpaired electron interacting with magnetic nuclei in the molecule is used as a sensitive probe for the electronic structure and its symmetry properties [6].

Many more examples of fascinating Möbius strip molecules of unique chemical stability against environmental perturbations are also discussed in Chapter 7. For instance, the interest in topology considerations for synthetic chemists and bioengineers has grown tremendously with the discovery of the "cyclotides," those cyclic disulfide-rich mini-protein ribbons isolated from certain plants. Their extraordinary chemical and biological stability is considered of paramount relevance in ongoing computer-aided medical drug design. The stability can be rationalized in terms of knot theory and topology invariants of the ribbon-shaped molecular fragments and the orientation of their π-orbitals.

The cyclotides fall into two main structural subfamilies: "Möbius cyclotides" contain a cis-proline in a loop that induces a local 180° backbone twist (like in a Möbius strip), whereas "Bracelet cyclotides" do not. Their tertiary structure is typically maintained by disulfide bridges. The cyclotides represent a new class of natural plant compounds considered ideal "template" molecules for drug design.

An extraordinary example is the cyclotide *Kalata B1* that adopts both Möbius strip and trefoil-knot topologies. Here, we give special recognition to David Craik from the University of Queensland, Australia. He was among the first scientists to discover the cyclotide family of stable globular circular proteins and pioneered their biomolecular synthesis. In traditional medicine in Central Africa, boiling water extracts of the local plant *kalata-kalata* were given as a tea to pregnant women to speed up childbirth. In 1995, the X-ray crystal structure of *Kalata B1*, the active constituent of the traditional tea, was solved and its cyclic structure with the cystine knot motif was elucidated [7]. The structural beauty of the new circular mini-proteins has also stimulated artists to create fascinating sculptures; a prominent example is Julian Voss-Andreae, born 1970 in Hamburg, Germany, see Fig. 9).

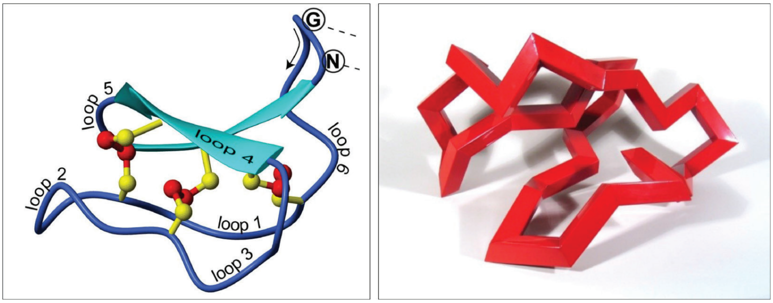


Figure 9 *Left*: Kalata B1 cyclotide, the glycine (G) and asparagine (N) amino acids are the terminal residues that are linked in a peptide bond to cyclize the peptide, i.e., give it a circular backbone [7]. *Right*: Julian Voss-Andreae, *Kalata* (2002). From his early works inspired by proteins, the molecular building blocks of life. (Source: Wikimedia Commons). For Julian Voss-Andreae, the main idea underlying his sculptures is the analogy between the technique of mitered cuts and protein folding. The sculptures offer a sensual experience of a world of science that is usually accessible only through the intellect [8].

Chapter 8 is devoted to topology applications from physics. It focuses on a novel class of solid-state material, the "graphenes," and the "topological insulators." In recent years, the study of band-structure topology has gained popularity after the discovery of materials that, at first glance, appear to be traditional band insulators, but when examined more closely turn out to be of a topologically different character. For these materials, a simple, continuous deformation of the band structure does not influence the topological invariants and, therefore, cannot change the band structure into that of a trivial insulator. The topological insulator phase in graphene was theoretically predicted in 2005 [9]. The behavior of the quantum spin state is topologically distinct from all previously known states of matter [10].

Recently, graphene nanoribbons (GNRs) have emerged as promising quasi-one-dimensional materials for next-generation electronic nanodevices. Because of the effect of edges and quantum confinement, GNRs can exhibit band gaps for transistor operation with exceptional switching speed and high carrier mobility. Particularly, GNRs with zigzag edges are expected to host edge-localized states, which may serve as key elements for graphene-based electronics and spintronics.

An interesting question arises as to what would happen to the quantum properties of the material when a zigzag-edged GNR torus transforms into a zigzag-edged Möbius GNR, causing the two zigzag edges to become one (see Fig. 10 *Right*). Answers to this question are also discussed in Chapter 8.

Some of the highlights with regard to fundamental issues of symmetry and topology in the arts and architecture are touched in Chapter 9. Large fractions of the chapter are devoted to fascinating examples of modern sculptures, houses, office buildings, and bridges inspired by Möbius strip topology.

But before continuing with our introductory remarks, we realized that it is time for a breathing pause for reflections:

When apparently in danger of turning the Introduction into an essay about why it is so important to know the history of topology in general and of the Möbius strip topology in particular, we remembered the famous quote by Antoine de Saint-Exupéry:

> "*If you want to build a ship, don't drum up the men to gather wood, and don't assign them tasks and work, but rather teach them to yearn for the vast and endless sea.*"

And so we stop continuing the Introduction.

Have we caught your interest? Just glance through the book, and we hope you will get stuck and start reading!

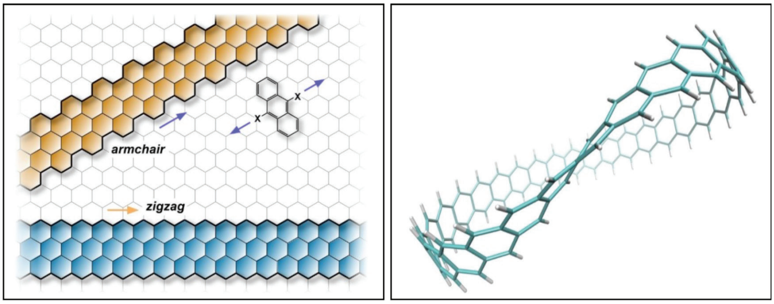


Figure 10 *Left*: Types of edges in graphene nanoribbon (GNR). Armchair GNRs are nonmagnetic semiconductors with the band gap determined by the width of the ribbon; zigzag GNRs are magnetic semiconductors with spin-polarized antiferromagnetic states at the edge (Source: Cheap Tubes Inc. (2019, September 26). A Guide to Graphene. AZoNano. Retrieved on March 10, 2021 from https://www.azonano.com/article.aspx?ArticleID=4841). *Right*: Structure of Möbius 30-cyclacene, which contains 30 fused benzene rings forming a closed loop with a single half-twist [11].

Appendix: The situation of the historic sources for the biographical Chapters 2 and 3

The overall source situation for the biographical part of this book on the life and work of A. F. Möbius is quite unfavorable, as was documented by the historic dissertation of André Loh from Leipzig University, 1994 [1]. Today it is very difficult to find original documents on the life and work of A. F. Möbius at all. The main reason for this is the destruction of the "Möbius Archive" of the Saxon Academy of Sciences in Leipzig. It burned to ashes during Allied bombing in December 1943. For the biographical Chapters 2 and 3 of our book, André Loh's dissertation was the main source of references. In the "Möbius Archive," all available documents by or about Möbius had been collected since 1885 by Leipzig scholars, of whom Curt Reinhardt (1855–1940) and Heinrich Liebmann (1874–1939) were of particular distinction; they had saved manuscripts from the Möbius's legacy, letters, diaries, documents, etc. Unfortunately, in most cases, not only the originals themselves but also their contents were lost because copies of only a few documents were made and stored elsewhere, for example a few letters between A. F. Möbius and C. F. Gauss and Alexander von Humboldt.

Because of this precarious source situation, André Loh tried to locate as many other repositories of documents by or about A. F. Möbius as possible, e.g., at the main stations of his life, such as the former *Fürstenschule Schulpforta*, where Möbius spent his childhood and school years, the universities of Leipzig, Göttingen, and Halle-Wittenberg, where Möbius spent his university student years. The archive searching there was mostly without success. The source situation concerning Möbius's professional life is much better. In the Leipzig University Archives, for example, there are still the personnel files and other holdings of the Faculty of Philosophy and the Leipzig Observatory, respectively, which refer to A. F. Möbius—and which André Loh was able to consult.

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