Local Gradient Theory for Dielectrics

Fundamentals and Applications

Olha Hrytsyna | Vasyl Kondrat

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To the brilliant memory of
Yaroslav Burak,
a famous Ukrainian scientist

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Preface

Experimental studies in the second half of the twentieth century revealed some phenomena and effects that cannot be appropriately described within the classical theory. Among such problems, we can mention the near-surface and interface inhomogeneity of electromechanical fields, size effects of mechanical and electrical characteristics of a material, nonlinear dependence of the inverse capacitance of thin dielectric films on their thickness, high-frequency dispersion of longitudinal elastic waves, propagation of antiplane surface shear waves (SH waves) in isotropic solids, linear response of polarization of centrosymmetric cubic crystals to the temperature gradient (thermopolarization effect) as well as to the stress gradient (flexoelectric effect), the emergence of a bound electric charge on the free surfaces of dielectric bodies, etc. (Abazari et al. 2015; Axe et al. 1970; Boukai et al. 2008; Bullen and Bolt 1985; Catalan et al. 2004, 2005; Kraut 1971; Li et al. 2003; Ma and Cross 2001a; Mead 1961; Rafikov and Savinov 1994; Tagantsev 1987; Tang and Alici 2011a, 2011b; Zubko et al. 2007). Solutions to these problems call for the development of new generalized mathematical models of dielectrics that take into account the inhomogeneity of the state of physically small elements of the body and describe their physical properties more scrupulously and accurately.

An extension of the classical field theory toward the abovementioned mathematical models became possible due to an intensive development of new technologies, in particular, nanotechnologies. Here we should mention an extensive utilization and design of new composite and porous materials, including nanocomposite and nanoporous ones, the engineering of microscale/nanoscale structures, nanoelectromechanical devices, sensors, and actuators. In many cases, such theories allow for avoiding the singularities in solutions to problems with dislocations, cracks, line sources, point loads and charges, etc.

There are several approaches to constructing extended theories of thermoelastic polarized solids. One group of theories considers the additional degrees of freedom (i.e., microrotations, microdeformations, etc.) for material points in order to take into account the contribution of the microstructure changes to the macroscopic behavior of a body. In such a way, since the 1960s, the fundamentals of micromorphic, microstretch, micropolar continua theories of dielectrics have developed (Dixon and Eringen 1965a, 1965b; Demiray and Eringen 1973; Eringen 1999, 2004; Lee et al. 2004). Nonlocal and gradient-type theories compose another group of extended theories of dielectrics. The nonlocal field theory for piezoelectricity with functional constitutive relations was proposed by Eringen (1984, 2002) and Eringen and Kim (1977). The gradient-type theories of dielectrics were developed by allowing the stored energy density to depend on the gradient of some physical quantities, namely, the strain tensor gradient (Kogan 1964), the polarization gradient (Mindlin 1968), or the electric field gradient (Landau and Lifshitz 1984; Yang et al. 2004). Note that the latter theory is similar to the so-called theory with electric quadrupoles because the electric field gradient is a thermodynamic conjugate of the electric quadrupole (Kafadar 1971).

In 1987, Burak proposed a new continuum-thermodynamic approach to the construction of a gradient-type theory of thermoelastic solids. The mentioned approach is based on taking account of non-diffusive and non-convective mass fluxes associated with the changes in the material microstructure. These fluxes were related to the process referred to as the local mass displacement (Burak 1987). By employing this approach, papers (Burak et al. 2007, 2008; Hrytsyna 2017a; Hrytsyna and Kondrat 2018; Hrytsyna and Moroz 2019; Kondrat and Hrytsyna 2012a) present the foundations of a gradient-type theory of the deformation of electrothermoelastic non-ferromagnetic polarized medium. This theory was called the local gradient theory of dielectrics. It is based on the accounting for the local mass displacement and its effect on mechanical, heat, and electromagnetic fields. The present book is concerned with the mathematical and physical aspects of the local gradient theory of dielectrics and its applications.

The book consists of five chapters. A short overview of generalized continuum theories of dielectric media taking account of the nonlocal effects is given in the first chapter. This chapter contains a brief description of the well-known and the most common approaches to the development of such theories within the framework of the continuum description.

In the second chapter, the fundamental concepts and basic relations of the local gradient electrothermomechanics of non-ferromagnetic dielectric solid bodies are formulated. It is shown that a gradient-type theory of dielectrics can be formulated by considering the contribution of the mass fluxes caused by changes in the material microstructure. In order to describe the process of local mass displacement, we introduce the corresponding physical quantities and obtain the balance-type equation to which these quantities are subordinated. It is shown that due to the local mass displacement, the gradient-type constitutive relations are obtained. A complete set of equations that include the balance equations, respective physical and geometric relations, as well as the corresponding boundary and jump conditions are formulated. The connection of the constructed theory with some generalized theories of dielectrics is analyzed. It is shown that for the developed theory of dielectrics, the principle of conformity is fulfilled. It means that in the limiting case of neglecting the local mass displacement, the obtained equations coincide with the equations of the classical theory.

In the third chapter, the local gradient theory of dielectrics is generalized by taking into account (i) the tensor-like representation of the parameters related to the local mass displacement, (ii) the irreversibility and inertia of the polarization and local mass displacement, (iii) the rheological properties of a dielectric medium with fading memory, and (iv) the electric quadrupoles. This, in particular, enabled us to obtain a dynamically coupled set of equations of local gradient thermomechanics of polarized medium.

The mathematical models of local gradient electrothermomechanics of non-ferromagnetic polarized solids that are developed in the second and third chapters have become the basis for theoretical studies of near-surface inhomogeneity of coupled fields in dielectrics, for the description of size effects, wave processes, etc. The mentioned investigations compose the fourth and fifth chapters of the book. In these chapters, it is shown that by taking the local mass displacement, its irreversibility and inertia into consideration, the classical continuum theory of thermoelastic dielectrics is extended to accommodate electromechanical interaction in centrosymmetric materials. The theories of polarized solids generalized in such a way make it possible to study the transition modes of the formation of near-surface inhomogeneity of coupled fields in dielectric bodies as well as to investigate the perturbation of mechanical, thermal, and electromagnetic fields due to the effect of rapidly changeable loads. Within the linear approximation, these theories describe a number of experimentally observed phenomena, including the surface, size, flexoelectric, pyroelectric, and thermopolarization effects in isotropic media, anomalous dependence of the capacitance of thin dielectric films on their thickness, the dispersive properties of polarized media, etc. Note that the above phenomena are not explained within the framework of classical theory of dielectrics.

The book is based on the results obtained by the authors over the last 20 years. It should be noted that a certain part of the basic results of this book has been published in a series of papers (Burak et al. 2007, 2008; Burak and Hrytsyna 2011; Chapla et al. 2009; Hrytsyna 2008, 2010, 2011, 2013a, b, c, 2014, 2015, 2016, 2017a, b; Hrytsyna and Kondrat 2018; Hrytsyna and Moroz 2019; Kondrat and Hrytsyna 2009a, b, c, 2010a, b, c, 2011, 2012a, b, c, 2018) as well as presented at a number of international conferences.

In conclusion, we would like to express our gratitude to the people who supported and helped us throughout this project. We would like to express our appreciation to Prof. Yaroslav Burak, our mentor, who initiated these studies and persistently encouraged us to do above researches. He had a great influence on the formation of scientific judgments of both authors of this book.

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