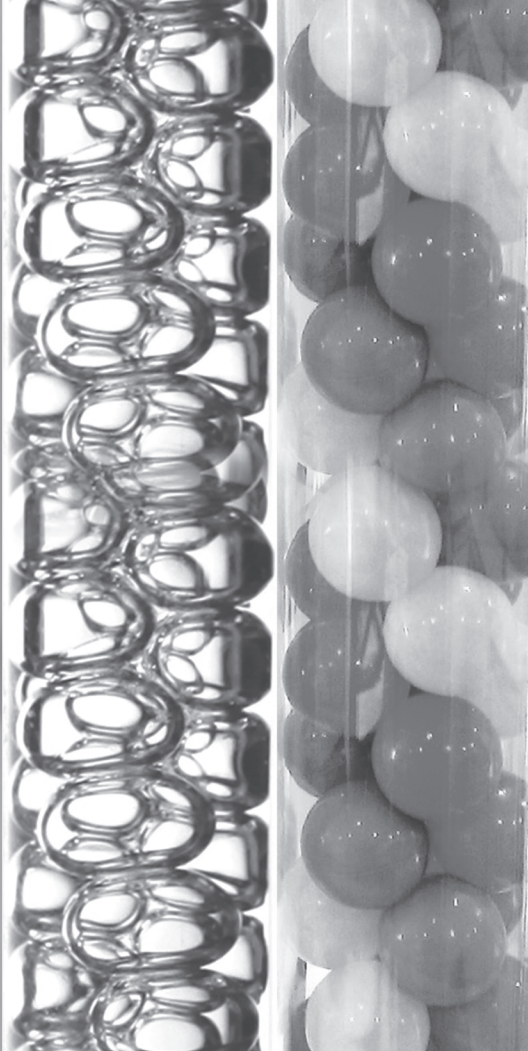


Columnar Structures of Spheres



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Fundamentals and Applications

Jens Winkelmann

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This book is dedicated to Prof. Denis Weaire, Prof. Stefan Hutzler,
and Dr. Adil Mughal, who introduced us to the fascinating field of
packing problems during our wonderful times at
Trinity College Dublin.

And Happy 80th Birthday to Denis!

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Preface

Columnar structures, many of which are helical, refer to dense cylindrical packings of particles. They are ubiquitous, for example they exist in the contexts of botany, foams, and nanoscience. There have been in-depth investigations of columnar structures of both hard spheres (e.g. ball bearings) and soft spheres (e.g. wet foams) through computer simulations, analytic derivations, or simple experiments. This monograph serves as a comprehensive guide for any scientist, engineer, or artist who would like to have a good grasp of the fundamentals and applications of such aesthetically appealing structures for his or her own professional interests.

This monograph is organized as follows: We first give an introduction to the field of packing problems, where such problems are not only related to the columnar structures presented in this monograph but also to the structures of condensed matter systems in general. We then discuss what columnar structures of spheres are, with an overview of their classifications and possible applications. This is followed by a discussion of the models and concepts employed in the study of such columnar structures. Following this, we discuss in detail a method of sequential deposition for generating columnar structures of hard spheres computationally or experimentally. We then present findings on columnar structures of soft spheres and on buckled columnar structures of longitudinally compressed hard-sphere chains.

This monograph is a collection of original research carried out by the two of us in the Foams and Complex Systems Research Group of Trinity College Dublin at respectively different eras [Ho-Kei Chan, post-doctoral research fellow (2009–2012); Jens Winkelmann, PhD student (2015–2020)] under the supervision of Prof. Denis Weaire and Prof. Stefan Hutzler and in collaboration with Dr. Adil Mughal

from Aberystwyth University. We are grateful to Denis, Stefan, and Adil for introducing us to the fascinating field of packing problems during our wonderful times at Trinity College Dublin. We end this preface by sharing a few memorable pictures taken during our times at Trinity. May the Foams and Complex Systems Research Group of Trinity College Dublin continue to thrive for many years to come.

Jens Winkelmann
Ho-Kei Chan
Autumn 2022